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On Optimum Power Allocation for Multi-Antenna Wideband Helicopter-to-Ground Communications

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On Optimum Power Allocation for Multi-Antenna Wideband Helicopter-to-Ground Communications

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***Abstract*—This paper introduces a generalized version of time-reversed, space-time block codes (TR-STBCs), called GTR-STBCs that operates with non-equal power allocation between two inter-symbol interference (ISI) channels in a 2 transmit, 1 receive antenna helicopter-to-ground radio link. The power allocation is parameterized by ρ, the portion of the total available power allocated to channel 1. The criteria for selecting the optimum ρ is minimizing the residual mean-squared error at the MMSE equalizer output. GTR-STBC is applied to measured channel impulse responses and a simple statistical channel model. The results show that 1) the optimum value of ρ gives the best tradeoff between signal-to-noise ratio and ISI; 2) equal power allocation may not be the optimum power allocation when channel side information is available; and 3) the optimum profile of ρ over measured channels channels is significantly different than from that in statistical channel models.**

I. Introduction

Wideband air-to-ground communication channels experience frequency selective fading characterized by severe inter-symbol interference (ISI), especially when the airborne transmitter is at a low altitude. Helicopter-to-ground communications present a particularly challenging scenario because the airborne can not only fly at a very low altitude, but also hover in particularly bad locations. Size, weight, and power limitations on most airborne platforms demand the use of RF power amplifiers operating at near or full saturation. Accompanying spectral limitations tend to push these systems to use single carrier modulations with, in the case of linear modulations, constellation points producing favorable peak-to-average ratios (e.g. MPSK or APSK).

It is well known that multiple antenna systems are capable of increasing reliability or throughput in multipath fading channels. In flat fading, the optimum signaling approach depends on what the transmitter knows [1]. If the transmitter knows the channels between each transmit and receive antenna, then spatio-temporal coding [2] is optimum in that it maximizes signal-to-noise ratio [1]. If the transmitter does not have this knowledge, then a diversity-maximizing orthogonal design (such as the Alamouti code [3]) is optimum [1]. In frequency selective fading, the general approach is to use OFDM and apply these techniques on a per subcarrier basis. Given the constraints described above, OFDM is often of limited interest in helicopter-to-ground communications.

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In the case of multi-antenna systems employing single carrier modulation and operating on frequency selective channels, time-reversed space-time block codes (TR-STBCs) [4]–[8] play the role that the Alamouti code does in frequency non-selective fading. When the transmitter knows the channels, the situation becomes less clear. Given the limitations imposed on the constellation points imposed by the peak-to-average ratio constraints, the use of arbitrary signals is not possible. In fact, other than the ability to switch between a small number of constellations, the only other variable available to the system is the power allocated to each transmit antenna.

In an effort increase the robustness of helicopter-to-ground communications in frequency selective fading, the use of multiple transmit antennas has been explored [9], [10]. Recent results applying TR-STBCs to multi-antenna channel impulse responses measured on a helicopter-to-ground link revealed some curious behavior [10]. An example of this curious behavior is shown in Fig. 1. This plot compares the simulated bit error rate (BER) performance of TR-STBC using MMSE equalizers to the simulated BER performance of a link using only *one* of the two available channels. TR-STBC performs better than the single-channel link using only channel 2, but *worse* than the single-channel link using only channel 1. Clearly, channel 1 is better, in some sense, than channel 2. In fact, channel 1 is so much better than channel 2 that incorporating channel 2 into a TR-STBC system only makes things worse.

In other words, there are cases where it is better to abandon traditional TR-STBC and use only one of the two available channels. The fact that this curious behavior *can* occur on real channels prompts one to ask, “Can this curious behavior be derived from the given impulse responses of two channels?” In other words, it appears that with some form of channel state information, it is possible to achieve better performance than TR-STBC.

This paper answers the question. As a conceptual tool, we consider a fixed-power transmitter that allocates a portion of the fixed power to each channel. This power allocation is parameterized by ρ, the proportion of total power allocated to channel 1. Using only one of the two available channels is captured by the case $\rho = 1$ (channel 1 only) or $\rho = 0$ (channel

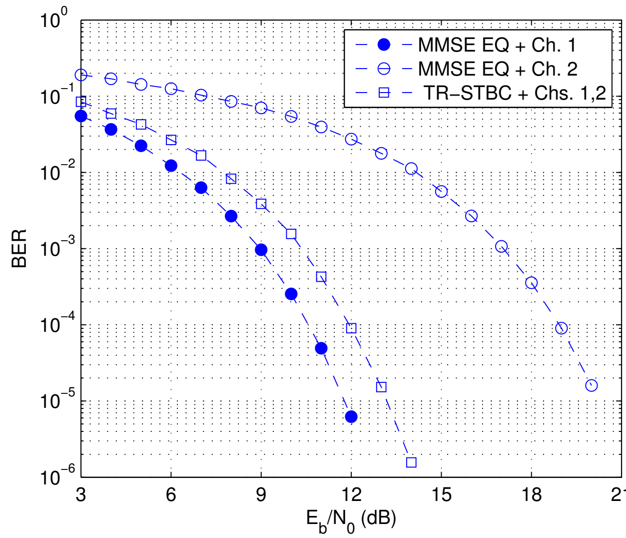

Fig. 1. Simulated BER plots for 20-Mbit/s QPSK using MMSE equalizers over a pair of representative impulse responses measured over a helicopter-go-ground channel. The circular markers are the BER results using a single antenna, the square markers are the BER results using both channels with TR-STBC. Reproduced from [10].

2 only). Next, we introduce Generalized TR-STBC (GTR-STBC), a modified version of TR-STBC that incorporates the unequal power allocation ρ. Equipped with GTR-STBC, we use the mean-squared error criterion to identify the optimum ρ. The motivations for the mean-squared error criterion are the following:

1) Mathematically tractable expressions for the mean-squared error at the equalizer output are easily derived [see (24) below].
2) The mean-square error criterion neatly captures the contributions of both ISI and additive noise at the equalizer output.
3) Generally speaking, reductions in mean-squared error lead to reductions in bit error rate.

By expressing the residual mean-squared error at the equalizer output as a function of ρ, we are able to chose ρ to minimize the residual mean-squared error for a given pair of channel impulse responses.

Finally, we apply the concepts to the set of measured channel impulse responses used in [10] to see if the mean-squared error criterion is capable of identifying the cases where the curious behavior occurs. The results show that the mean-squared error criterion does indeed capture the curious behavior.

The conceptual tool of unequal power allocation together with GTR-STBC define a simple transmit diversity scheme based on the partial knowledge of the channel by the transmitter. Here, the transmitter only needs to know ρ, which is easy to compute at the receiver and send back to the transmitter. This simple scheme includes transmit selection diversity ($\rho = 0$ or 1) and traditional TR-STBC ($\rho = 1/2$).

II. Generalized TR-STBC (GTR-STBC): Non-Equal Power Allocations Using TR-STBC

An abstraction (to the symbol level) for a 2×1 GTR-STBC system is illustrated in Fig. 2. Here the system transmits the symbol sequence $a(0), \ldots, a(2N-1)$ over two transmit antennas to one receive antenna. The equivalent discrete-time channel between transmit antenna 1 and the receive antenna is represented by the impulse response $h_1(n)$ for $-M_1 \leq n \leq N_1$ whereas the equivalent discrete-time channel between transmit antenna 2 and the receive antenna is represented by the impulse response $h_2(n)$ for $-M_2 \leq n \leq N_2$.

The GTR-STBC encoder partitions the symbol sequence $a(0), \ldots, a(2N-1)$ into two sequences $a_1(n)$ and $a_2(n)$ as shown in Fig. 2. The length-$2N$ packet is transmitted in two intervals[1] each spanning N symbol intervals. During the first interval $a_1(0), \ldots, a_1(N-1)$ is transmitted from antenna 1 whereas $a_2(0), \ldots, a_2(N-1)$ is transmitted from antenna 2. During the second interval, $a_2^*(N-1), \ldots, a_2^*(0)$ is transmitted from antenna 1 whereas $-a_1^*(N-1), \ldots, -a_1^*(0)$ is transmitted from antenna 2.

Power division using $0 \leq \rho \leq 1$ is accomplished by the GTR-STBC system along the lines illustrated in Fig. 2. Amplitude scaling is applied to the signals entering each channel so as to divide the power between the channels. Here ρ represents the proportion of total power allocated to transmit antenna 1. The traditional TR-STBC system is a special case[2] for which $\rho = 1/2$. The square-root is used in Fig. 2 because the amplitudes are what are being modified—the energy (or power) is the square of the amplitude.

The received signal $x(n)$ is given by

$$x(n) = \sqrt{\rho} s_1(n) * h_1(n) + \sqrt{1-\rho} s_2(n) * h_2(n) + w(n) \quad (1)$$

where $w(n)$ is a complex-valued Gaussian random sequence with zero mean and autocovariance function

$$\mathrm{E}\left\{ w(n) w^*(n-k) \right\} = 2\sigma_w^2 \delta(k). \quad (2)$$

The GTR-STBC decoder partitions $x(n)$ into $x_1(n)$ and $x_2(n)$ as follows:

$$\begin{aligned} x_1(n) &= x(n) \text{ for } 0 \leq n \leq N-1 \\ x_2(n-N) &= x(n) \text{ for } N \leq n \leq 2N-1. \end{aligned} \quad (3)$$

These two sequences are given by

$$\begin{aligned} x_1(n) = {} & \sqrt{\rho}\, a_1(n) * h_1(n) \\ & + \sqrt{1-\rho}\, a_2(n) * h_2(n) + w_1(n) \end{aligned} \quad (4)$$

$$\begin{aligned} x_2(n) = {} & \sqrt{\rho}\, a_2^*(-n) * h_1(n) \\ & - \sqrt{1-\rho}\, a_1^*(-n) * h_2(n) + w_2(n) \end{aligned} \quad (5)$$

[1] In a practical implementation, a guard interval at least as long as the longest channel impulse response must be inserted between the two intervals. Here, such an interval is assumed, although we won't complicate the notation to make this explicit.

[2] In the traditional TR-STBC system, $\rho = 1/2$ is not included in the development nor the notation because the same power is assumed to be applied to each channel. Hence there is no need to account for it, other than in normalizing the noise variance.

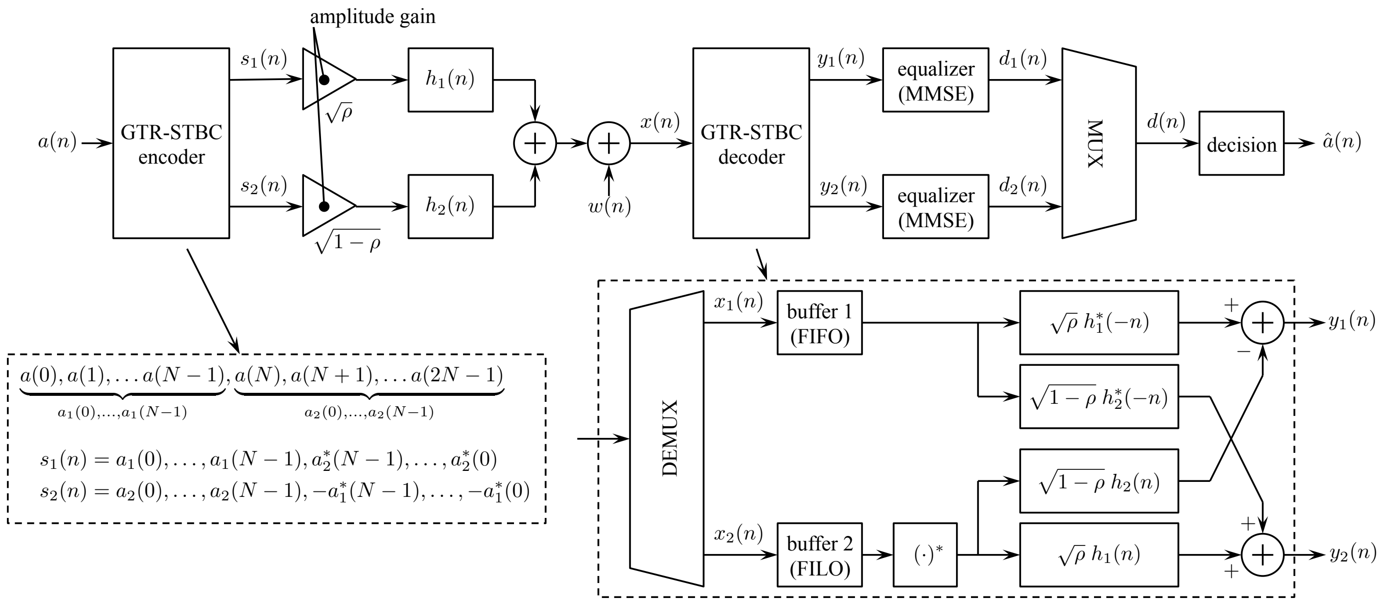

Fig. 2. A block diagram of the GTR-STBC system: TR-SRBC with unequal power allocation using $0 \le \rho \le 1$.

where $w_1(n)$ and $w_2(n)$ are related to $w(n)$ the same way $x_1(n)$ and $x_2(n)$ are related to $x(n)$. The TR-STBC decoder processes $x_1(n)$ and $x_2(n)$ using a bank of filters based on the channel impulse responses $h_1(n)$ and $h_2(n)$ as shown. The result of this processing is a pair of parallel sequences $y_1(n)$ and $y_2(n)$ which may be expressed as

$$\begin{aligned} y_1(n) &= x_1(n) * \sqrt{\rho}\, h_1^*(-n) - x_2^*(-n) * \sqrt{1-\rho}\, h_2(n) \\ &= a_1(n) * \underbrace{\Big[\rho\, h_1(n) * h_1^*(-n) + (1-\rho)\, h_2(n) * h_2^*(-n)\Big]}_{h_{\text{eq}}(n)} \\ &\quad + \underbrace{w_1(n) * \sqrt{\rho}\, h_1^*(-n) + w_2^*(-n) * \sqrt{1-\rho}\, h_2(n)}_{v_1(n)} \end{aligned} \tag{6}$$

$$\begin{aligned} y_2(n) &= x_1(n) * \sqrt{1-\rho}\, h_2^*(-n) + x_2^*(-n) * \sqrt{\rho}\, h_1(n) \\ &= a_2(n) * \underbrace{\Big[(1-\rho)\, h_2(n) * h_2^*(-n) + \rho\, h_1^*(-n) * h_1(n)\Big]}_{h_{\text{eq}}(n)} \\ &\quad + \underbrace{w_1(n) * \sqrt{1-\rho}\, h_2^*(-n) + w_2^*(-n) * \sqrt{\rho}\, h_1(n)}_{v_2(n)}. \end{aligned} \tag{7}$$

These equations show that the equivalent composite channel for non-equal power allocation is

$$h_{\text{eq}}(n) = \rho\ \underbrace{h_1(n) * h_1^*(-n)}_{\eta_1(n)} + (1-\rho)\ \underbrace{h_2(n) * h_2^*(-n)}_{\eta_2(n)}. \tag{8}$$

Because the support for $h_1(n)$ is $-M_1 \le n \le N_1$, the support for $\eta_1(n)$ is $-(M_1+N_1) \le n \le (M_1+N_1)$. Similarly because the support for $h_2(n)$ is $-M_2 \le n \le N_2$, the support for $\eta_2(n)$ is $-(M_2+N_2) \le n \le (M_2+N_2)$. Consequently, the support for $h_{\text{eq}}(n)$ is $-N_{\text{eq}} \le n \le N_{\text{eq}}$ where

$$N_{\text{eq}} = \max\Big\{M_1 + N_1, M_2 + N_2\Big\}. \tag{9}$$

The noise sequences $v_1(n)$ and $v_2(n)$ are complex-valued Gaussian random sequences each with zero mean and auto-correlation and cross correlation functions

$$\mathrm{E}\Big\{v_1(n)v_1^*(n-k)\Big\} = \mathrm{E}\Big\{v_2(n)v_2^*(n-k)\Big\} = 2\sigma_w^2 h_{\text{eq}}(k). \tag{10}$$

$$\mathrm{E}\Big\{v_1(n)v_2^*(n-k)\Big\} = 0. \tag{11}$$

By way of summary, the TR-STBC system presents to the equalizers the sequences $y_1(n)$ and $y_2(n)$ which may be represented by

$$y_1(n) = a_1(n) * h_{\text{eq}}(n) + v_1(n) \tag{12}$$

$$y_2(n) = a_2(n) * h_{\text{eq}}(n) + v_2(n) \tag{13}$$

where $h_{\text{eq}}(n)$ is given by (8). The noise terms $v_1(n)$ and $v_2(n)$ are uncorrelated zero-mean Gaussian random sequences each with autocorrelation function (10). A pair of equalizers operate in parallel on $y_1(n)$ and $y_2(n)$. Because the the noise sequences $v_1(n)$ and $v_2(n)$ are statistically equivalent and $h_{\text{eq}}(n)$ is common to both, the pair of equalizers operating on $y_1(n)$ and $y_2(n)$ are identical as long as $a_1(n)$ and $a_2(n)$ are statistically equivalent (the usual case). Any equalizer can be applied here (linear or non-linear, with or without noise whitening) with the usual performance-complexity tradeoffs. In the next section, we apply MMSE equalizers because MMSE equalizers permit a mathematically tractable analysis for the resulting mean-squared error. We leverage the analytical expression to find the value of ρ that minimizes the MMSE.

III. MMSE Equalization

We assume that the MMSE equalizer is a length-$(2L+1)$ FIR filter with coefficients

$$c(-L), \dots, c(0), \dots, c(L).$$

The equalizer output $d_i(n)$, for $i = 1, 2$ is

$$d_i(n) = \sum_{\ell=-L}^{L} c(k) y_i(n-\ell) \tag{14}$$

and the corresponding error is

$$e_i(n) = a_i(n) - d_i(n). \tag{15}$$

Assuming a symbol-spaced equalizer and uncorrelated data, the vector of filter coefficients $\mathbf{c}_{\text{opt}}$ that minimizes the mean-squared error is

$$\mathbf{c}_{\text{opt}} = \left[\mathbf{G}\mathbf{G}^\dagger + \frac{\sigma_w^2}{\sigma_a^2}\mathbf{H}_{\text{eq}}\right]^{-1} \mathbf{g} \tag{16}$$

where $\mathbf{G}$ is the $(2L+1) \times (2L+1+2N_{\text{eq}})$ matrix

$$\mathbf{G} = \begin{bmatrix} h_{\text{eq}}(N_{\text{eq}}) & \cdots & h(-N_{\text{eq}}) & & & \\ & h(N_{\text{eq}}) & \cdots & h(-N_{\text{eq}}) & & \\ & & & \ddots & & \\ & & & h(N_{\text{eq}}) & \cdots & h(-N_{\text{eq}}) \end{bmatrix};$$

$\mathbf{H}_{\text{eq}}$ is the $(2L+1) \times (2L+1)$ matrix

$$\mathbf{H}_{\text{eq}} = \begin{bmatrix} h_{\text{eq}}(0) & h_{\text{eq}}(-1) & \cdots & h_{\text{eq}}(-2L) \\ h_{\text{eq}}(1) & h_{\text{eq}}(0) & \cdots & h_{\text{eq}}(-2L+1) \\ \vdots & & & \\ h_{\text{eq}}(2L) & h_{\text{eq}}(2L-1) & \cdots & h_{\text{eq}}(0) \end{bmatrix}$$

where it is understood that $h_{\text{eq}}(n) = 0$ for $|n| > N_{\text{eq}}$; $\mathbf{g}$ is the $(2L+1) \times 1$ vector

$$\mathbf{g} = [\ \underbrace{0\ \cdots\ 0}_{L-N_{\text{eq}}\text{ zeros}}\ \ h_{\text{eq}}^*(N_{\text{eq}})\ \cdots\ h_{\text{eq}}^*(-N_{\text{eq}})\ \ \underbrace{0\ \cdots\ 0}_{L-N_{\text{eq}}\text{ zeros}}\ ]^\top$$

and

$$\sigma_a^2 = \frac{1}{2}\mathrm{E}\left\{|a(n)|^2\right\} \tag{17}$$

is the average symbol energy. The corresponding mean-squared error is

$$\mathcal{E} = 2\sigma_a^2 \left(1 - \mathbf{g}^\dagger \left[\mathbf{G}\mathbf{G}^\dagger + \frac{\sigma_w^2}{\sigma_a^2}\mathbf{H}_{\text{eq}}\right]^{-1} \mathbf{g}\right). \tag{18}$$

Using (8), it is straightforward to show that

$$\mathbf{G} = \rho\mathbf{G}_1 + (1-\rho)\mathbf{G}_2 \tag{19}$$

$$\mathbf{g} = \rho\mathbf{g}_1 + (1-\rho)\mathbf{g}_2 \tag{20}$$

$$\mathbf{H}_{\text{eq}} = \rho\mathbf{H}_1 + (1-\rho)\mathbf{H}_2 \tag{21}$$

where $\mathbf{G}_1$, $\mathbf{g}_1$, and $\mathbf{H}_1$ are formed from $\eta_1(n)$ the same way $\mathbf{G}$, $\mathbf{g}$, and $\mathbf{H}_{\text{eq}}$ are formed from $h_{\text{eq}}(n)$, respectively. Similar definitions apply to $\mathbf{G}_2$, $\mathbf{g}_2$, and $\mathbf{H}_2$ with $\eta_2(n)$.

Making the substitutions for $\mathbf{G}$, $\mathbf{g}$, and $\mathbf{H}_{\text{eq}}$ gives

$$\mathbf{c}_{\text{opt}} = \mathbf{M}^{-1}\left(\rho\mathbf{g}_1 + (1-\rho)\mathbf{g}_2\right) \tag{22}$$

where

$$\mathbf{M} = \left(\rho\mathbf{G}_1 + (1-\rho)\mathbf{G}_2\right)\left(\rho\mathbf{G}_1 + (1-\rho)\mathbf{G}_2\right)^\dagger + \frac{\sigma_w^2}{\sigma_a^2}\left(\rho\mathbf{H}_1 + (1-\rho)\mathbf{H}_2\right). \tag{23}$$

The mean-squared error is

$$\mathcal{E} = 2\sigma_a^2\left[1 - \left(\rho\mathbf{g}_1 + (1-\rho)\mathbf{g}_2\right)^\dagger \mathbf{M}^{-1}\left(\rho\mathbf{g}_1 + (1-\rho)\mathbf{g}_2\right)\right]. \tag{24}$$

Equation (24) is the desired relationship: for a given pair of channels $h_1(n)$ and $h_2(n)$, it expresses the mean-squared error at the output of the MMSE equalizer as a function of the power allocation ρ. Thus, for a fixed pair channels, one can choose the power allocation to minimize the achievable mean-squared error.

IV. Numerical Results

The forgoing analysis was applied to a helicopter-to-ground radio link using 39,300 channel impulse responses captured during the channel sounding experiments described in [9]. For the modulation, we use 20 Mbit/s QPSK with a square-root raised-cosine pulse shape with 50% excess bandwidth [11]. The matched filter output is sampled at 1 sample/symbol. The equivalent discrete-time channels are defined as the system with QPSK symbols at the input and the sampled matched filter outputs as the output. In the results shown below, $h_1(n)$ is the equivalent discrete-time channel between the nose antenna and the receive antenna and $h_2(n)$ is the the channel between the tail antenna and the receive antenna.

Two normalizations are applied to the channels: the *natural normalization* and the *equal-energy normalization*. Let $h_{1,\text{u}}(n)$ and $h_{2,\text{u}}(n)$ be unnormalized channel impulse responses for the two equivalent discrete-time channels obtained directly from the channel sounding data, and let

$$E_1 = \sum_{n=-M_1}^{N_1} |h_{1,\text{u}}(n)|^2 \qquad E_2 = \sum_{n=-M_2}^{N_2} |h_{2,\text{u}}(n)|^2 \tag{25}$$

be the energies in two channels. The natural normalization uses

$$h_1(n) = \frac{1}{\sqrt{E}} h_{1,\text{u}}(n) \qquad h_2(n) = \frac{1}{\sqrt{E}} h_{2,\text{u}}(n) \tag{26}$$

where $E = \max\{E_1, E_2\}$. This normalizes the stronger of the two channels to unit energy.[3] We call this the natural normalization because in real multi-antenna scenarios, especially those with antennas separated by several tens or even hundreds of wavelenghts, it is often the case that one of the channels is stronger than the other.

[3]The motivation for unit energy is for scaling the noise variance to define the signal-to-noise ratio.

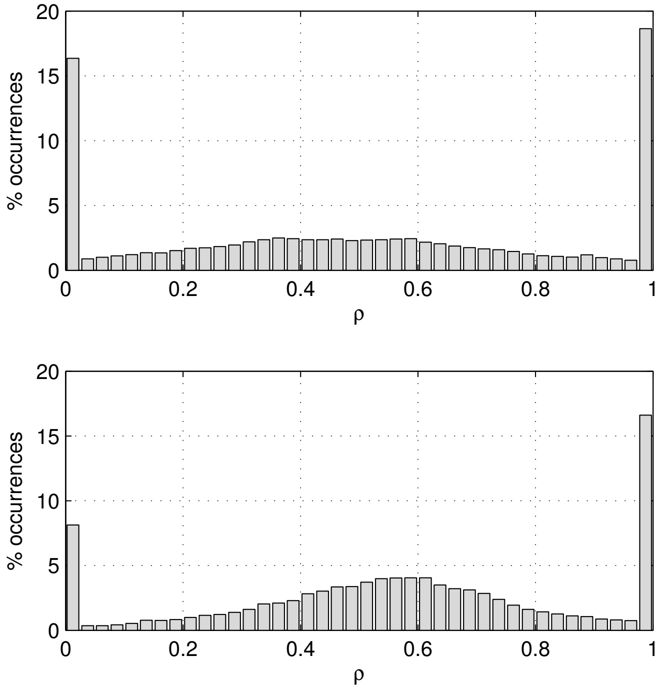

Fig. 3. Optimum power allocations in the mean-squared error sense for the measured channel impulse responses using the natural normalization: (top) $E_b/N_0 = 10$ dB; (bottom) $E_b/N_0 = 20$ dB.

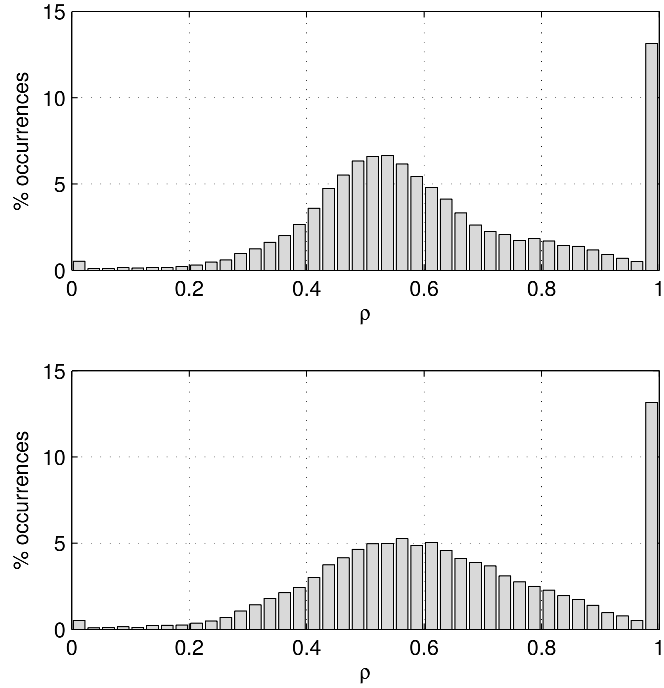

Fig. 4. Optimum power allocations in the mean-squared error sense for the measured channel impulse responses using the equal-energy normalization: (top) $E_b/N_0 = 10$ dB; (bottom) $E_b/N_0 = 20$ dB.

For the equal-energy normalization, we use

$$h_1(n) = \frac{1}{\sqrt{E_1}} h_{1,\mathrm{u}}(n) \qquad h_2(n) = \frac{1}{\sqrt{E_2}} h_{2,\mathrm{u}}(n). \tag{27}$$

Here, both channels are normalized to unit energy. This is more typical of statistical or mathematical models of of multi-antenna propagation.

The numerical results were produced as follows. For each of the 39,300 pairs of channel impulse responses, the impulse response were normalized using one of the two procedures described above. The value of ρ that minimizes (24) for $L = 3 \times N_{\mathrm{eq}}$ [12] was computed.

The results using the natural normalization are summarized by the histograms shown in Fig. 3. For $E_b/N_0 = 10$ dB [Fig. 3 (a)], approximately 35% of the channel pairs prefer the use of a single channel over the use of both channels. When E_b/N_0 is increased to 20 dB [Fig. 3 (b)], the number of channel pairs that prefer the use of a single channel falls to 25%. This is because as E_b/N_0 increases, ISI tends to dominate the contribution to residual mean-squared error so that the preference for applying some power to both channels increases.

The results for the equal-energy normalizations are shown in Fig. 4. Using this normalization, the signal-to-noise ratio penalty associated with the weaker channel is removed, and ISI tends to dominate the choice for ρ. For much of the run, $h_1(n)$, the propagation from the helicopter nose to the receive antenna forms a better channel from an ISI point of view. This manifests itself in Fig. 4 by a strong preference for $\rho \approx 1$ and almost no preference for $\rho \approx 0$.

It is interesting to compare these results with what might be inferred from using a simple statistical channel model. To do so, we use a simple Gaussian model for each channel such as that used in [13]. In this experiment, channel 1 comprises 11 IID zero-mean complex-valued Gaussian random variables and channel 2 comprises 18 IID zero-mean complex-valued Gaussian random variables. These numbers, 11 and 18, are the average lengths of $h_1(n)$ and $h_2(n)$, respectively, in our measured data set. The channels were normalized using the equal-energy normalization described above and 39,300 independent realizations were produced. The results are summarized by the histograms in Fig. 5. The temptation is to think of the optimum ρ as a normally distributed random variable, but it should be kept in mind that this is not the case because $0 \leq \rho \leq 1$. For modest values of E_b/N_0 we observe that the mean value of the optimum ρ is about 0.5. Given the fact that $\rho = 0.5$ corresponds to traditional TR-STBC, we see that the simple statistical model suggests that traditional TR-STBC is the best on average. This is in contrast to the conclusion drawn from the measured channel data, where a strong preference for transmit selection diversity is observed. As E_b/N_0 increases, the optimum the optimum value of ρ increases because the contribution to residual mean-squared error from additive noise decreases relative to the contribution from ISI. The optimum $\rho > 0.5$ means the system prefers to allocate more energy to channel 1 than channel 2. This makes sense because channel 1 is shorter, and this tends to contribute less residual ISI at the equalizer output.

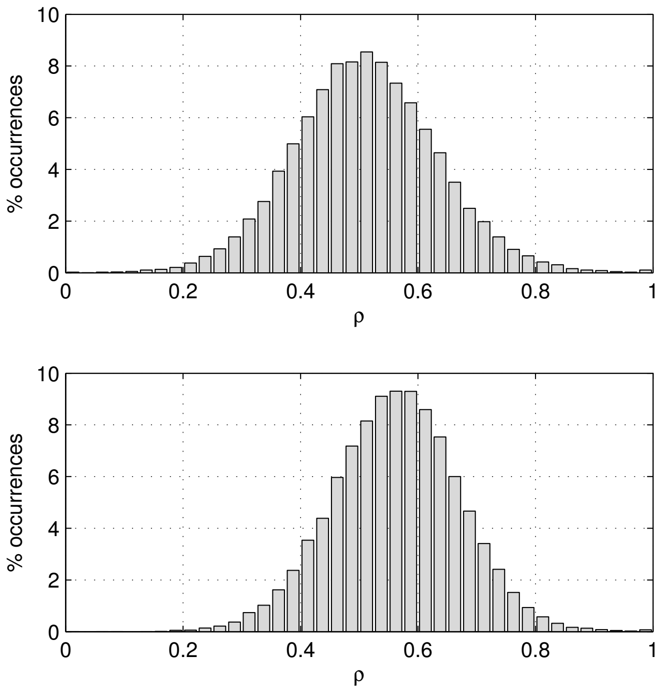

Fig. 5. Optimum power allocations in the mean-squared error sense for pairs of equal-energy channels where the channel coefficients are IID zero-mean complex-valued Gaussian random variables: (top) $E_b/N_0 = 10$ dB; (bottom) $E_b/N_0 = 20$ dB.

V. Conclusions

A criterion that allows one to predict when it is better to use transmit selection diversity (i.e., one transmit antenna) or the diversity achievable through TR-STBC (i.e., two transmit antennas) was developed. The criterion is the residual mean-squared error at the output of an MMSE equalizer. The residual mean-squared error was not only a mathematically tractable quantity, but also an excellent predictor of the curious behavior illustrated in Fig. 1.

These concepts were applied to a set of measured channel impulse responses collected from helicopter-to-ground channel sounding experiments. For each pair of channel impulse responses, the value of ρ that minimized the residual mean-squared error (24) was computed. The computed values for ρ were used to form histograms to summarize the results. These results illustrate the following points:

1) In a 2-transmit, 1-receive antenna system operating in a frequency non-selective fading environment, if the two channels have unequal gains, the optimum thing to do is apply all of the available power to the stronger channel. That is, transmit selection diversity is optimum. In contrast, on a frequency selective fading channel, the optimum approach is to apply power to produce the best trade-off between SNR and ISI. That is, transmit selection diversity may not be optimum. The optimum value of ρ associated with the GTR-STBC system described in this paper identifies the best trade-off between SNR and ISI. There are some channel pairs for which $\rho = 0$ or 1 is the optimum (transmit diversity case) and some channel pairs for which $\rho = 1/2$ (traditional TR-STBC). But there are many channel pairs for which neither of these is optimum.
2) On our measured channels, transmit selection diversity was more common than traditional TR-STBC.
3) On a statistical channel, such as the one used in [13], traditional TR-STBC is the best thing to do on average. This is in contrast to the results from the measured channels. Consequently, the optimum power allocation in a real setting is not predicted well by simple statistical channel models.

The expression for the residual mean-squared error (24) was developed by formulating a generalization of TR-STBC that included not only transmit selection diversity and traditional TR-STBC, but also a generalization that permitted the transmitter to allocate unequal power to each transmit antenna. This generalization, called generalized TR-STBC (GTR-STBC), can be used as the basis of transmit diversity system where the transmitter has partial channel state information (in the form of ρ). This transmit diversity system is apropos to single carrier modulations operating in situations with peak-to-average power ratio constraints.

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